Period 1B
1/22/08

Supercorrection Four-Point Form

Name:

#_6_ Convince me that you now understand the concept- connect to previous HW, notes, etc. Be sure to also explain the error(s) that you made.

Original _3_/4 Supercorrection _4_/4

Use the properties of logs to write the expression as a sum, difference and or multiple of logs. Simplify

$$\log \frac{100\sqrt{y}}{x^2}$$

When I looked at this problem, I recalled the log properties that would apply to it to be able to write it as a sum, difference and or multiple of logs. ($\log a + \log b = \log ab$, $\log a - \log b = \log \frac{a}{b}$) Both of these worked and I used them correctly. After using them, I had an expression that looked like this:

$$2 + \tfrac{1}{2}\log y - \log x^2$$

In the directions I was told to simplify the expression as much as possible, and I thought I had, but I overlooked the $\log x^2$ part. I forgot about the property that says $\log a^x = x \cdot \log a$. Using this property, the expression can be simplified further as $\boxed{2 + \tfrac{1}{2}\log y - 2\log x}$ I simply overlooked the last step in simplifying the expression.

Correct solution:

Use the properties of logs to write the expression as a sum, difference and or multiple of logs. Simplify.

$$\log \frac{100\sqrt{y}}{x^2}$$

$\log_2 a + \log_2 b = \log_2 ab$
$\log_2 a - \log_2 b = \log_2 \frac{a}{b}$
$\log_2 a^x = x \cdot \log_2 a$

3 logarithm properties

$$\log \frac{100\sqrt{y}}{x^2}$$
$$= \log 100 + \log \sqrt{y} - \log x^2$$
$$= \log 100 + \tfrac{1}{2}\log y - \log x^2$$
$$= 2 + \tfrac{1}{2}\log y - \log x^2$$
$$= 2 + \tfrac{1}{2}\log y - 2\log x$$
$$= \boxed{2 + \tfrac{1}{2}\log y - 2\log x}$$

#_9_ Convince me that you now understand the concept- connect to previous HW, notes, etc. Be sure to also explain the error(s) that you made.

Original _2_/4 Supercorrection _4_/4

Select values for a and b with $a > b$, such that $\log_b a$ is less than 0. Justify your choice for a and b.

1 Correct Answer $\log_{\frac{1}{2}} 2 = -1$

$a = 2$
$b = \frac{1}{2}$

When looking this problem now, it seems so easy. $\log_b a$ needs to be less than 0. ($\log_b a < 0$) If we substitute 0 for an actual negative number, like -1, then we have a solid start on the problem. ($\log_b a = -1$) So, $b^{-1} = a$. Since the domain of logarithms is only positive numbers, a and b must both remain positive. Going back to $b^{-1} = a$, we must remember that a must be greater than b. Therefore, b needs to be a number that when raised to the -1 power it increases and remains positive. This will be our answer for a. What better than a fraction! There are many was of getting a correct answer in this problem, but if we use $\frac{1}{2}$ as b, then a will equal 2. ($(\frac{1}{2})^{-1} = 2$) This satisfies our requirements for a, so this is a correct solution. I was confused with this problem at first and tried it, but decided to come back to it when I was finished with the rest of the test. If I had spent more time on the problem I probably would have solved it.

Correct solution:

Select values for a and b, with $a > b$, such that $\log_b a$ is less than 0. Justify your choice for a and b.

$\log_b a < 0$

pick a negative number

$\log_b a = -1$

$\therefore b^{-1} = a$

substitute values for b trying to find a value of a which is greater than b

$(-1)^{-1} = \frac{1}{(-1)^1} = \frac{1}{-1} = -1$ X

$3^{-1} = \frac{1}{3^1} = \frac{1}{3}$ X

$(\frac{1}{2})^{-1} = 2$ ✓

$(\frac{1}{4})^{-1} = 4$ ✓

When $b = \frac{1}{2}$ and $\frac{1}{4}$, $a = 2$ and 4 respectively. Therefore $\frac{1}{2}$ and $\frac{1}{4}$ are both one of many solutions to this problem.

$\boxed{a = 2}$

$\boxed{b = \frac{1}{2}}$

Good!

Supercorrection Four-Point Form Name:____________

11 a. Convince me that you now understand the concept- connect to previous HW, notes, etc. Be sure to also explain the error(s) that you made.

Original 3 /4 Supercorrection 4 /4

Given $\log_a b = 2.3219$, approximate

$\log_a \frac{1}{b^2} = y$

This problem looks very scary at first, but I attempted it and would have gotten it right, but I made one little mistake that threw off my answer. From the original logarithm, we can say that $a^{2.3219} = b$. We can then rewrite the equation for which we are trying to solve to $\log_a b^{-2} = y$. If we substitute $a^{2.3219}$ for b, then we get $\log_a (a^{2.3219})^{-2} = y$ This is what I forgot to include when I worked on this problem.* I also solved it a little differently to get my answer of -2.3219 whereas if I had to gotten to include the -2 in the way I have solved it here, I would have gotten 2.3219. Either way, in the end the answer would be -4.6438 because $(a^m)^n = a^{m \cdot n}$. Because of that simple, little mistake, I got the problem wrong and it should have been something I would have picked up on while going back over my test, but I didn't have enough time to do so unfortunately.

Correct solution:

Given $\log_a b = 2.3219$, approximate

$\log_a \frac{1}{b^2}$

$a^{2.3219} = b$

substitute $a^{2.3219}$ for b

$\log_a \frac{1}{b^2} = y$

$\log_a b^{-2} = y$

$\log_a (a^{2.3219})^{-2} = y$

$\therefore a^y = (a^{2.3219})^{-2}$

$a^y = a^{-4.6438}$

$\boxed{y = -4.6438}$

* From here, we can say that $a^y = (a^{2.3219})^{-2}$. Then, $a^y = a^{-4.6438}$. Now it is easy to see that y equals -4.6438.

Nice!

11 c. Convince me that you now understand the concept- connect to previous HW, notes, etc. Be sure to also explain the error(s) that you made.

Original 2 /4 Supercorrection ____/4

Given $\log_a b = 2.3219$, approximate

$\log_a ab$

When trying to solve this problem, for some reason I presumed a to equal 2.3219 the reason for which I have no idea... It seems really silly, but I honestly do not know what I was thinking. I probably had a mix-up with b being equal to $a^{2.3219}$. Anyway, if I had had more time to go back over my test, I would have recognized the logarithm property of $\log_a a + \log_a b = \log_a a \cdot b$. Using this property, we can rewrite $\log_a ab = y$ as $\log_a a + \log_a b = y$. Another logarithm property states that $\log_a a = 1$. Using this property, we can rewrite the last equation as $1 + \log_a b = y$. From the directions we know that $\log_a b = 2.3219$, so we can substitute that for $\log_a b$. We know have $1 + 2.3219 = y$ as our equation which is simple to solve. $3.3219 = y$. Easy easy easy!

Correct solution:

Given $\log_a b = 2.3219$, approximate

$\log_a ab$

$= \log_a ab = y$

$= \log_a a + \log_a b = y$

$= 1$ $\quad = 2.3219$

$= 1 + 2.3219 = y$

$= \boxed{3.3219 = y}$

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Supercorrection Four-Point Form Name:

#_12_ Convince me that you now understand the concept- connect to previous HW, notes, etc. Be sure to also explain the error(s) that you made.
Original _2_ /4 Supercorrection _4_ /4

Solve for x.

$x^{\frac{3}{4}} = 8$

This was the very last problem I attempted on the test and I attempted it with maybe 2 minutes left of class, so I was just looking to get something down (hopefully the right answer) and then go back and rework it during supercorrections. I was in such a hurry that I didn't really think of the best way to isolate x. I just went with my first thought of taking the ln of both sides so that I could bring down the exponent using the rule that states: $\log_a b^x = x \cdot \log_a b$. This seemed reasonable because then I could just divide both sides by $\frac{3}{4}$ and be close to an answer. From here, however, everything just fell apart as I was pressed for time. I attempted to do things to both sides of the equation that I wasn't sure were possible. All I had to do was raise both sides of the equation to the $\frac{4}{3}$ power. This would cancel out the $\frac{3}{4}$ exponent x was being raised to. What would be left would be: $x = 8^{\frac{4}{3}}$ which is easy to solve once we change 8 to 2^3. We would have: $x = (2^3)^{\frac{4}{3}}$ which equals $2^{\frac{12}{3}}$ which equals 2^4 which equals 16. Yay! Problem solved. $x = 16$

Correct solution:

Solve for x.

$x^{\frac{3}{4}} = 8$

$x^{\frac{3}{4}} = 8$

$(x^{\frac{3}{4}})^{\frac{4}{3}} = 8^{\frac{4}{3}}$

$x^{\frac{12}{12}} = 8^{\frac{4}{3}}$

$x = 8^{\frac{4}{3}}$

$x = (2^3)^{\frac{4}{3}}$

$x = 2^{\frac{12}{3}}$

$x = 2^4$

$\boxed{x = 16}$

Beautiful!

Your way would have worked; you just made an error in eliminating the ln. The inverse of $\ln x$ is e^x...

#_____ Convince me that you now understand the concept- connect to previous HW, notes, etc. Be sure to also explain the error(s) that you made.
Original ____/4 Supercorrection ____/4

Correct solution:

Period 1B
1/18/08

ALGEBRA 2 HON $\frac{96}{104} = 92\%$ Name:

A Finely Crafted O'Brien Unit 5 Test

Calculator Section: You may use a calculator. Show all work and circle your answer. Use your time wisely; you will be able to earn additional credit after the timed portion of the test by completing Supercorrections. When you finish, put away your calculator and you come up to get the non-calculator part- you may continue to work on both sections without your calculator.

1. In the following statements, $a > 0,\ a \neq 1$ and $x > 0,\ y > 0$. Determine whether each is true or false.

a. $\log_a a = 1$
$a^1 = a$ True

b. $\log_a 1 = a$
$a^a = 1$ False

c. $\log_a 0 = 1$
$a^1 = a$ False

d. $\log_a 1 = 0$
$a^0 = 1$ True

e. $\log_a xy = \log_a x + \log_a y$ $\log 1000 = \log 10 + \log 100$
True

f. $\log_a(x+y) = \log_a x + \log_a y$ $\log(2+3) = \log 2 + \log 3$
$\log 5 = \log 2 + \log 3$
False

g. $\log_a(x+y) = \log_a x \cdot \log_a y$
False

h. $\log_a xy = \log_a x \cdot \log_a y$ $\log_4 2 \cdot 3 = \log_4 2 \cdot \log_a 3$
$\log_4 6 = \log_4 2 \cdot \log_4 3$
$1.29 = 0.52 \cdot 0.79$
$1.29 \neq 0.41$
False

i. $\log_a x^y = y \log_a x$
True

j. $\log_a x^y = (\log_a x)^y$
False

2. Find the smallest value of x for which $3^x \geq 1{,}000{,}000$.

$3^x \geq 1{,}000{,}000$
$\ln 3^x \geq \ln 1{,}000{,}000$
$x \cdot \ln 3 \geq \ln 1{,}000{,}000$
$\frac{x \cdot \ln 3}{\ln 3} \geq \frac{\ln 1{,}000{,}000}{\ln 3}$
$\boxed{x \geq \frac{\ln 1{,}000{,}000}{\ln 3}}$

$\boxed{\text{Smallest value of } x = \frac{\ln 1{,}000{,}000}{\ln 3}}$

3. The number of bacteria B in a culture increases according to the equation $B = B_0 e^{kt}$. There were 400 bacteria at time $t = 0$ and 900 bacteria at time $t = 4$ hours. Will there be 1900 bacteria after 12 hours? If not, how many will there be?

$t = 0$
$B = 400$

$t = 4$
$B = 900$

$400 = B_0 e^{kt}$
$400 = B_0 e^{k \cdot 0}$
$400 = B_0 e^0$
$400 = B_0$

$900 = 400 e^{4k}$
$\frac{900}{400} = \frac{400 e^{4k}}{400}$
$\frac{9}{4} = e^{4k}$
$\ln \frac{9}{4} = \ln e^{4k}$
$\ln \frac{9}{4} = 4k$

$\frac{\ln \frac{9}{4}}{4} = k$

$400 = B_0 e^{k \cdot 0}$
$400 = B_0 e^0$
$400 = B_0$

$B = 400 e^{12 \cdot \frac{\ln \frac{9}{4}}{4}}$

$\boxed{B = 4{,}556.\cancel{25} \text{ after 12 hours}}$

$900 = 400 e^{4k}$
$\frac{900}{400} = \frac{400 e^{4k}}{400}$
$\frac{9}{4} = e^{4k}$
$\ln \frac{9}{4} = \ln e^{4k}$

$\ln \frac{9}{4} = 4k$
$\frac{\ln \frac{9}{4}}{4} = k$

$B = 400 e^{12 \cdot \frac{\ln \frac{9}{4}}{4}}$
$B =$

4. At the right is a "solution" to the equation $100 = 18e^{4k}$.

$\log_e(18e^{4k})$

$100 = 18e^{4k}$

$\ln 100 = \ln(18e^{4k})$ Cannot distribute logs

$\ln 100 = \ln 18 \ln(e^{4k})$

$\frac{\ln 100}{\ln 18} = \ln(e^{4k})$

$\frac{\ln 100}{\ln 18} = 4k$

$\frac{\ln 100}{4\ln 18} = k$

$k \approx 0.398$

$\frac{\ln 100}{\ln}$

a. Check the answer $k \approx 0.398$ back in the equation $100 = 18e^{4k}$ and show that it doesn't work.

$100 = 18e^{4 \cdot 0.398}$

$100 \neq 88.44$

b. Circle the error in the "solution" and give a correct solution below. 1.59

$100 = 18e^{4k}$

$\frac{100}{18} = \frac{18e^{4k}}{18}$

$\frac{100}{18} = e^{4k}$

$\ln \frac{100}{18} = \ln e^{4k}$

$\ln \frac{100}{18} = 4k$

$\frac{\ln \frac{100}{18}}{4} = \frac{4k}{4}$

$\frac{\ln \frac{100}{18}}{4} = k$

$\boxed{0.43 \approx k}$

$100 = 18e^{4k}$

$\ln 100 = \ln 18e^{4k}$

$\ln 100 = \ln 18e^{4k}$

$\ln 100 = 4k \cdot \ln 18$

$\frac{\ln 100}{\ln 18} = \frac{4k \cdot \ln 18}{\ln 18}$

$\frac{\ln 100}{\ln 18} = 4k$

$100 = 18e^{4k}$

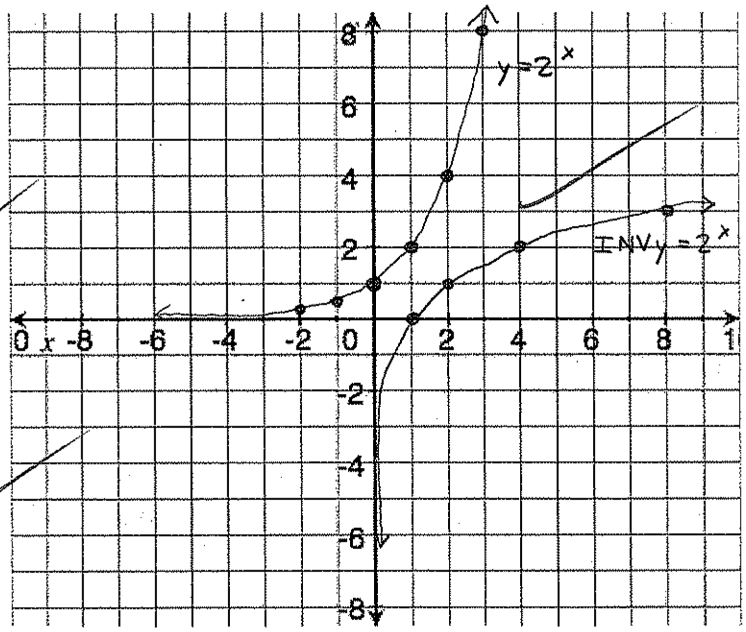
$\ln 100 = \ln 18e^{4k}$

$\ln 100 = 18^{4k}$

5.

a. Make a table and accurately graph $y = 2^x$.

x	-3	-2	-1	0	1	2	3	4
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	16



b. Make a table of values for the inverse of $y = 2^x$.

x	-2	-1	0	1	2	4	8
y	/	/	/	0	1	2	3

$x = 2^y$ $\quad \log_2 x = y$ $\quad 2^y = -2$

c. Sketch the graph of the inverse of $y = 2^x$. (Label which graph is which.)

d. Write the equation of the inverse of $y = 2^x$ in $y =$ form.

$y = 2^x$

$x = 2^y$

$\boxed{y = \log_2 x}$

$y = 2^x$

$x = 2^y$

$\log_2 x = y$

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6. Use the properties of logs to write the expression as a sum, difference, and/or multiple of logs. Simplify where possible.

$$\log \frac{100\sqrt{y}}{x^2}$$

$\log 100 + \log \sqrt{y} - \log x^2$

$= \log 100 + \frac{1}{2}\log y - \log x^2$

$= \boxed{2 + \frac{1}{2}\log y - \log x^2}$

$\log 100\sqrt{y} - \log x^2$

$\log \frac{100\sqrt{y}}{x^2}$

7

So close!

7. Write the expression as the logarithm of a single quantity.

$$\ln 3 + \frac{1}{3}\ln(4 - x^2) - \ln x$$

$= \ln 3 + \ln \sqrt[3]{(4-x^2)} - \ln x$

$= \ln 3\left(\sqrt[3]{4-x^2}\right) - \ln x$

$= \boxed{\ln \frac{3\left(\sqrt[3]{4-x^2}\right)}{x}}$

8. Molly needs to find the range of $f(x) = \frac{e^x + 7}{2}$. She has a brilliant idea that she can use the inverse of $f(x)$ in a clever way to do this. Find $f^{-1}(x)$ and use it to find the range of $f(x)$.

$f(x) = \frac{e^x+7}{2}$

$y = \frac{e^x+7}{2}$

$x = \frac{e^y+7}{2}$

$2x = e^y + 7$

$2x - 7 = e^y$

$\ln 2x - 7 = \ln e^y$

$\boxed{\ln(2x-7) = y}$

~~Domain: $x > \frac{7}{2}$~~

$\log_e 2x - 7 = y$

$2x - 7 > 0$

$2x > 7$

$\frac{2x}{2} > \frac{7}{2}$

$x > \frac{7}{2}$

$\boxed{\text{Range of } f(x) = \frac{e^x+7}{2} = \{f(x) = f(x) > \frac{7}{2}\}}$

9. Select values for a and b, with $a > b$, such that $\log_b a$ is less than zero. Justify your choice for a and b.

5

$\log_b a < 0$

$\log_2 4 < -\frac{1}{2}$

$(2)^{-\frac{1}{2}} = 4$

$\log_{\frac{1}{2}} 2 = -1$

$(2)^{-3} = \frac{1}{2^3}$

$\left(\frac{1}{4}\right)^2 = \frac{1}{16}$

$\left(\frac{1}{4}\right)^{-2}$

$\left(\frac{1}{4}\right)^2$ $\frac{1}{16}$ 16

$\left(\frac{1}{2}\right)^{-1}$ $\frac{1}{\left(\frac{2}{1}\right)^1} = \frac{1}{2}$

$\log_b a \leq 0$

$\log_{\frac{1}{2}} 2 = -1$

$\left(\frac{1}{2}\right)^{-1}$

$= 2$

$x^{\frac{3}{4}} = 8$

$\log_{\frac{3}{4}} x^{\frac{3}{4}} = \log_{\frac{3}{4}} 8$

$x = \log_{\frac{3}{4}} 8$

$x = \frac{\log 8}{\log \frac{3}{4}}$

10. Solve for x.

$$\log_a x + \log_a(x-2) = \log_a(x+4)$$

$\log_a x(x-2) = \log_a (x+4)$

$\log_a x^2-2x = \log_a x+4$

$a^{\log_a x^2-2x} = a^{\log_a x+4}$

$x^2-2x = x+4$

$x^2-3x = 4$

$x^2-3x-4=0$

$(x-4)(x+1)=0$

$x^2+x-4x-4=0$

$x^2-3x-4=0$

$\boxed{x=4}$ or $x=-1$

6

11. Given that $\log_a b \approx 2.3219$, approximate

$a^{2.3219} = b$ $\boxed{y = \frac{1}{2.3219}}$

a. $\log_a \frac{1}{b^2} = y$

$a^{2.3219} = b$

$a^y = \frac{1}{b^2}$

$a^y = b^{-2}$

$\log_a \frac{1}{a^{2.3219}}$

$\log_a a^{-2.3219}$

$= \boxed{-2.3219}$

b. $\log_b a$

$\log_b 2.3219 = y$

$b^y = 2.3219$

$(a^{2.3219})^y = 2.3219$

$a^{2.3219y} = 2.3219$

$\ln a^{2.3219y} = \ln 2.3219$

$2.3219y \cdot \ln a = \ln 2.3219$

$2.3219y = \frac{\ln 2.3219}{\ln a}$

$2.3219y = \frac{\ln 2.3219}{\ln 2.3219}$

$2.3219y = 1$

c. $\log_a ab$

$\log_{2.3219}$ $2.3219 \cdot a^{2.3219}$

$= a^{2.3219}$

$= \boxed{b}$

6

12. Solve for x.

$$x^{\frac{3}{4}} = 8$$

$\ln x^{\frac{3}{4}} = \ln 8$

$\frac{3}{4} \cdot \ln x = \ln 8$

$\frac{\frac{3}{4} \cdot \ln x}{\frac{3}{4}} = \frac{\ln 8}{\frac{3}{4}}$

$\ln x = \frac{\ln 8}{\frac{3}{4}}$ Correct

$\ln^{\ln x} = \ln^{\left(\frac{\ln 8}{\frac{3}{4}}\right)}$

$e^\wedge$

$\boxed{x = \ln\left(\frac{\ln 8}{\frac{3}{4}}\right)}$

$x = 16$

$x^{\frac{3}{4}} = 8$

$(x^{\frac{3}{4}})^{\frac{4}{3}} = 8^{\frac{4}{3}}$

$x = 8^{\frac{4}{3}}$

$x = (2^3)^{\frac{4}{3}}$

$x = 2^{\frac{12}{3}}$

$x = 2^4$

$x = 16$

13. Suppose $g(x) = \log_7 x$ and $h(x) = 7^x$.

a. Find $g(h(x))$.

$g(7^x)$

$g(7^x) = \log_7 7^x$

$= \boxed{x}$

b. Find $h(g(x))$.

$h(\log_7 x)$

$h(\log_7 x) = 7^{\log_7 x}$

$= \boxed{x}$

c. What is the relationship between the functions g and h?

They are inverses

Bonus: Choose values for *a, b,* and *c* so that the equation below is true. Justify your choice of *a, b,* and *c*.

$$\log_c[\log_a(\log_b c)] = 0$$

$\log_a \frac{1}{b^2} = y$

$\log_a b^{-2} = y$

$-2\log_a b = y$

$-2(2.3219) = y$

$\log_a ab$

$= \log_a a + \log_a b$

$= 1 + 2.3219$